8. Klasse Übungsaufgaben

Rechnen mit Bruchtermen

1. Bestimme die Definitionsmenge:

   (a) \( \frac{5x^2 - a}{36x^2 - 16x} \)

   (b) \( \frac{1}{x - 6} - \frac{1}{6x + 1} \)

2. Vereinliche:

   (a) \( \frac{45x - 20}{36x^2 - 16x} \)

   (b) \( \frac{(ab)^2}{a^3b - a^2b^3} \)

3. Bringe auf einen Bruchstrich: \( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_1 + R_2} \)

4. Vereinliche:

   (a) \( \frac{1}{x - 1} - \frac{1}{x + 2} \)

   (b) \( \frac{24 - x}{x + 3} - 8 \)

   (c) \( \frac{6x^2 + 5}{36x^2 - 16x} + \frac{3x}{8 - 18x} \)

   (d) \( \frac{6x + 11}{2x + 4} - \frac{2x + 5}{x^2 + 2x - 3} \)

5. Vereinliche:

   (a) \( \frac{74x - 34}{x + 1} \cdot \frac{x^2 + 1}{74x + 34} \)

   (b) \( \frac{az}{z - a} : \frac{a^2z^3}{az - a^2} \)

6. Vereinliche:

   (a) \( \frac{J}{C} \)

   (b) \( \frac{J}{T} \)

   (c) \( \frac{1 \overline{x} - 2}{1 - \frac{1}{x}} \)
8. Klasse Lösungen

Rechnen mit Bruchtermen

1. (a) Nebenrechnung: \(36x^2 - 16x = 0;\)  
   \[4x(9x - 4) = 0;\]  
   \(x = 0\) oder \(9x - 4 = 0;\)  
   also verboten: \(x = 0\) oder \(x = 4/9;\)  
   Somit \(D = \mathbb{Q} \setminus \{-\frac{1}{6}, 6\}\)  

   Damit ist \(D = \mathbb{Q} \setminus \{0; \frac{4}{9}\}\)

2. (a) \(\frac{45x - 20}{36x^2 - 16x} = \frac{5(9x - 4)}{4x(9x - 4)} = \frac{5}{4x}\)
   (b) \(\frac{(ab)^2}{a^2b - a^2b^3} = \frac{a^2b^2}{a^2b(a - b^2)} = \frac{b}{a - b^2}\)

3. \(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_1 + R_2} = \frac{R_2(R_1 + R_2) + R_1(R_1 + R_2) + R_1 R_2}{R_1 R_2 (R_1 + R_2)} = \frac{3R_1 R_2 + R_1^2 + R_2^2}{R_1 R_2 (R_1 + R_2)}\)

4. (a) \(\frac{1}{x-1} - \frac{1}{x+2} = \frac{x+2}{(x-1)(x+2)} - \frac{x-1}{(x+1)(x+2)} = \frac{x+2- (x-1)}{(x-1)(x+2)} = \frac{x+2-(x-1)}{(x+1)(x+2)} = \frac{3}{(x+1)(x+2)}\)
   (b) \(\frac{24-x}{x+3} - 8 = \frac{24-x-8(x+3)}{x+3} = \frac{24-x-8x-24}{x+3} = \frac{24-8x+3}{x+3} = \frac{-9x}{x+3}\)
   (c) Zuerst faktorisieren, dann \((-1)\)-Trick, dann auf gleichen Nenner bringen:
      \(\frac{6x^2+5}{36x^2-16} + \frac{3x}{8-18x} = \frac{6x^2+5}{4x(9x-4)} + \frac{3x}{2(4-9x)} = \frac{6x^2+5}{4x(9x-4)} - \frac{3x}{2(9x-4)} = \frac{6x^2+5-3x}{4x(9x-4)} = \frac{5}{4x(9x-4)}\)
      \(\frac{2x+5}{x^2+2x} - 3 = \frac{2x+5}{x(x+2)} - 3 = \frac{(2x+5)x}{x(x+2)} = \frac{2x+5}{x+2}\)
      \(\frac{2x+5}{x+2} - 3 = \frac{2x+5}{x+2} - \frac{3(x+2)}{x+2} = \frac{2x+5-3x-6}{x+2} = \frac{-x-1}{x+2}\)
   (d) \(\frac{6x^2+11x-3}{2x+1} = \frac{6x^2+11x-12x+12x-3}{2x+1} = \frac{6x^2+11x-12x+12x-3}{2x+1} = \frac{6x^2-12x+12x-3}{2x+1} = \frac{-5x+2}{2x+1}\)

5. (a) \(\frac{74x - 34}{x+1} \cdot \frac{x^2 + 1}{74x + 34} = \frac{x^2 + 1}{x+1}\)
   (b) \(\frac{az}{z-a} : \frac{a^2z^3}{az - a^2} = \frac{az}{z-a} \cdot \frac{az - a^2}{a^2z^3} = \frac{az \cdot a(z-a)}{(z-a) \cdot a^2z^3} = \frac{1}{z^2}\)

6. (a) \(\frac{J}{C} = \frac{J}{C} : J = \frac{J}{C \cdot J} = \frac{1}{C}\)
   (b) \(\frac{J}{C} = J \cdot \frac{C}{J} = \frac{J \cdot C}{J} = \frac{C}{1} = C\)
   (c) \(\frac{1-x}{x^2} \cdot x - 1 = \frac{1-2x^2}{x^2} \cdot x = \frac{1-2x^2}{x^2} \cdot x = \frac{(1-2x^2)x}{x^2(x-1)} = \frac{1-2x^2}{x^2(x-1)}\)