

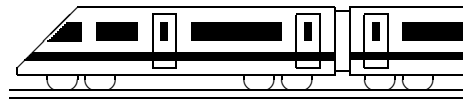


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|---|-----------|
| Übungsaufgaben weitere Themen (alter LP) | W |
| Kurvendiskussion: Musteraufgabe | 09 |

Fertigen Sie vollständige Kurvendiskussionen an:

1. $\frac{x}{x^2 + 1}$

2. $\frac{1}{x} - x^2$



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|---|-----------|
| Lösungen weitere Themen (alter LP) | W |
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1.

$D = \mathbb{R}$

A: $\lim_{x \rightarrow \pm\infty} f(x) = 0$:

Waagrechte Asymptote $y = 0$

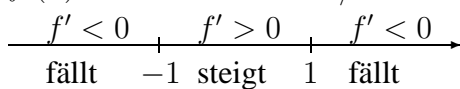
S: $f(-x) = \frac{-x}{(-x)^2+1} = -\frac{x}{x^2+1} = -f(x)$:

Punktsymmetrie zum Ursprung

N: $f(x) = 0: x = 0$

E: $f'(x) = \frac{(x^2+1) \cdot 1 - x \cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$

$f'(x) = 0: 1 - x^2 = 0; x_{1/2} = \pm 1.$



Min $(-1 | -\frac{1}{2})$ Max $(1 | \frac{1}{2})$

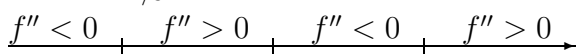
W: $f''(x) = \frac{(x^2+1)^2 \cdot (-2x) - (1-x^2) \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4} =$

$\frac{(x^2+1) \cdot (-2x) - (1-x^2) \cdot 4x}{(x^2+1)^3} = \frac{2x^3 - 6x}{(x^2+1)^3}$

$f''(x) = 0: 2x^3 - 6x = 0;$

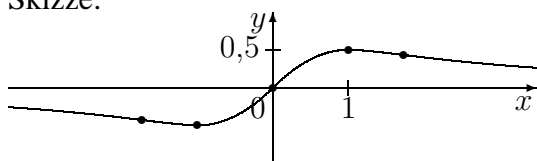
$2x(x^2 - 3) = 0;$

$x_1 = 0, x_{2/3} = \pm\sqrt{3}.$



$(-\sqrt{3} | -\frac{\sqrt{3}}{4})$ $(0|0)$ $(\sqrt{3} | \frac{\sqrt{3}}{4})$

Skizze:



$W_f = [-\frac{1}{2}; \frac{1}{2}]$

2.

Verschiedene Schreibweisen für den Funktionsterm:

$f(x) = \frac{1}{x} - x^2 = \frac{1-x^3}{x} = x^{-1} - x^2$

$D = \mathbb{R} \setminus \{0\}$

A: $\lim_{x \rightarrow \pm\infty} f(x) \rightarrow -\infty$

” +1 ”
 $\lim_{x \rightarrow \pm 0} f(x) = \frac{\pm 1}{\pm 0} \rightarrow \pm\infty$

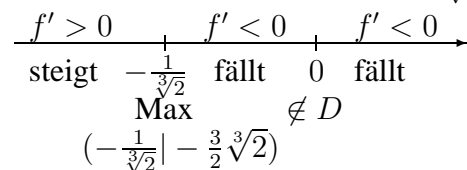
(Senkr. As., Pol 1. Ordnung $x = 0$)

S: Keine Symmetrie

N: $f(x) = 0: 1 - x^3 = 0; x = 1$

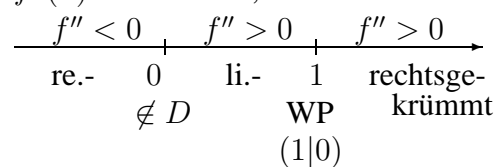
E: $f'(x) = -x^{-2} - 2x = \frac{-1-2x^3}{x^2}$

$f'(x) = 0: -1 - 2x^3 = 0; x = -\frac{1}{\sqrt[3]{2}}$

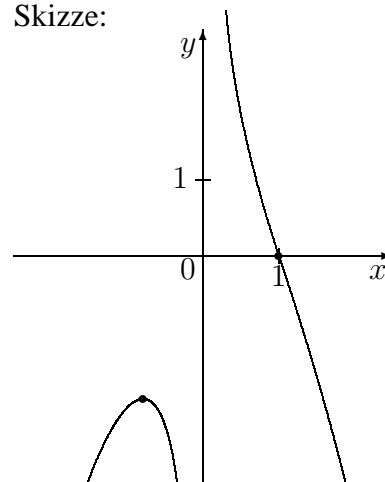


W: $f''(x) = +2x^{-3} - 2 = \frac{2}{x^3} - 2$

$f''(x) = 0: x^3 = 1; x = 1$



Skizze:



$W_f = \mathbb{R}$